A Supplementary material

A.1 Alternative partitions

The $S_{CP}$ distributions with alternative partitions are shown in Fig. 6 for $D^0 \rightarrow K^- K^+ \pi^- \pi^+$ decays and Fig. 7 for $D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$ decays.

The $S_{CP}$ distributions with alternative partitions for the $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ control channel are shown in Fig. 8, for combined magnet up and magnet down data.

![Figure 6](image1.png)

Figure 6: Distribution of $S_{CP}$ for $D^0 \rightarrow K^- K^+ \pi^- \pi^+$ decays with (a) 16 bins and (b) 64 bins. The points show the data distribution and the solid line is a reference Gaussian distribution corresponding to the no CPV hypothesis.

![Figure 7](image2.png)

Figure 7: Distribution of $S_{CP}$ for $D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$ decays with (a) 64 bins and (b) 256 bins. The points show the data distribution and the solid line is a reference Gaussian distribution corresponding to the no CPV hypothesis.
Figure 8: Distribution of $S_{CP}$ for $D^0 \to K^\mp \pi^\mp \pi^\mp \pi^\pm$ decays with (a) 16 bins and (b) 1024 bins. The points show the data distribution and the solid line is a reference Gaussian distribution corresponding to the no CPV hypothesis.

A.2 Bin definitions

The phase-space partitions for $D^0 \to K^- K^+ \pi^- \pi^+$ and $D^0 \to \pi^- \pi^+ \pi^+ \pi^-$ decays are defined in terms of five invariant mass-squared combinations of the $D^0$ meson decay products. The definitions of default partitions in terms of these five variables are shown along with the $S_{CP}$ value for each corresponding bin for $D^0 \to K^- K^+ \pi^- \pi^+$ decays and $D^0 \to \pi^- \pi^+ \pi^+ \pi^-$ decays in Fig. 9 and Fig. 10, respectively.
Figure 9: Definitions of the default partition of 32 bins across the five-dimensional phase space of the $D^0 \to K^- K^+ \pi^- \pi^+$ decay. The vertical lines show the range covered in the given invariant mass-squared combination in units of GeV$^2$/c$^4$. The invariant mass-squared combinations $s(1,2)$, $s(2,3)$, $s(1,2,3)$, $s(2,3,4)$, and $s(3,4)$ correspond to $s(K^-, K^+)$, $s(K^+, \pi^-)$, $s(K^-, K^+, \pi^-)$, $s(K^+, \pi^-, \pi^+)$, and $s(\pi^-, \pi^+)$, respectively. The markers on the first plot show the value of $S_{CP}$ for each corresponding bin.
Figure 10: Definitions of the default partition of 128 bins across the five-dimensional phase space of the $D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$ decay. The vertical lines show the range covered in the given invariant mass-squared combination in units of $\text{GeV}^2/c^4$. The invariant mass-squared combinations $s(1,2)$, $s(2,3)$, $s(1,2,3)$, $s(2,3,4)$, and $s(3,4)$ correspond to $s(\pi^- , \pi^+)$, $s(\pi^+ , \pi^-)$, $s(\pi^- , \pi^+ , \pi^+)$, $s(\pi^+ , \pi^+ , \pi^-)$, and $s(\pi^+ , \pi^-)$, respectively. The markers on the first plot show the value of $S_{CP}$ for each corresponding bin.