7 Supplementary material: visualization of the $CP$ asymmetry

The time-dependent decay widths in Eq. (1) can be combined into a $CP$ asymmetry explicitly dependent on $m_{hh}$ and decay angles as

$$\frac{\Gamma(t) - \Gamma(t)}{\Gamma(t) + \Gamma(t)} = S(m_{hh}, \Omega) \sin(\Delta m dt) - C(m_{hh}, \Omega) \cos(\Delta m dt),$$

(15)

where

$$S(m_{hh}, \Omega) = \frac{2I m(A^*A)}{|A|^2 + |A|^2}, \quad \text{and} \quad C(m_{hh}, \Omega) = \frac{|A|^2 - |A|^2}{|A|^2 + |A|^2}. \quad \text{(16)}$$

Since the asymmetry depends on the location in the $(m_{hh}, \Omega)$ phase space, the overall asymmetry integrated over the phase space is diluted as both $S$ and $C$ change sign. It is also further diluted by other experimental effects, e.g. wrong flavour tagging. In order to view the time dependent asymmetry, we transform the event-by-event decay time by changing $t$ to $t' = t + dt(m_{hh}, \Omega)$ by using

$$\cos[\Delta m dt(m_{hh}, \Omega)] = \frac{S(m_{hh}, \Omega)}{\sqrt{S(m_{hh}, \Omega)^2 + C(m_{hh}, \Omega)^2}}$$

$$\sin[\Delta m dt(m_{hh}, \Omega)] = -\frac{C(m_{hh}, \Omega)}{\sqrt{S(m_{hh}, \Omega)^2 + C(m_{hh}, \Omega)^2}}.$$

(17)

where $S(m_{hh}, \Omega)$ and $C(m_{hh}, \Omega)$ are determined by the fit to the data. The asymmetry in Eq. (15) is transformed to a single sine function with positive coefficient:

$$\frac{\Gamma(t') - \Gamma(t')}{\Gamma(t') + \Gamma(t')} = \sqrt{S(m_{hh}, \Omega)^2 + C(m_{hh}, \Omega)^2} \sin(\Delta m dt').$$

(18)

The new asymmetry is not diluted by its location in phase space because only positive coefficients are summed. The quantity $\Delta m dt'$ is taken to be modulo of $2\pi$. The transformation only depends on the value of $(m_{hh}, \Omega)$, not the decay time $t$.

To obtain the data asymmetry distribution as function of $t'$, we first calculate $t'$ event-by-event for the data, the pseudo-experimental signal and background samples using the Fit 1 result. The pseudo-experimental samples are generated according to their PDFs used in the fit. The tagged $B^0$ and $\bar{B}^0$ data $t'$ distributions are subtracted by the corresponding background distributions, then asymmetries in bins of $t'$ are calculated. The red curve is the expectation from Fit 1, obtained from the pseudo-experimental signal asymmetry distribution. The $CP$ asymmetries for the sum of all resonant components are shown for the decay time in Fig. 7 and for the shifted decay time in Fig. 8. In the latter case the time modulation of the $CP$ asymmetry is clearly seen.
Figure 7: CP asymmetry as a function of decay time for all components in $B^0 \rightarrow J/\psi \pi^+ \pi^-$. 

Figure 8: CP asymmetry as function of shifted decay time for all components in $B^0 \rightarrow J/\psi \pi^+ \pi^-$. 

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