Figure 1: Distribution of the $\mu^+\mu^-$ mass versus the $\pi^+\pi^-\mu^+\mu^-$ mass, before the vetoes to remove dimuon resonances. The black lines define the regions to veto $\phi \rightarrow \mu^+\mu^-$, $J/\psi \rightarrow \mu^+\mu^-$, and $\psi(2S) \rightarrow \mu^+\mu^-$ decays; they are 1.010–1.030 GeV/$c^2$, 2.796–3.216 GeV/$c^2$, and 3.436–3.806 GeV/$c^2$, respectively.
Figure 2: Distribution of the $\mu^+\mu^-$ mass in the range 2–4.5 GeV/c$^2$ to show the $J/\psi$ and $\psi(2S)$ resonances with their radiative tails. All candidates with $\pi^+\pi^-\mu^+\mu^-$ invariant mass in the range 5.19–6.99 GeV/c$^2$ are considered. The red lines define the regions to veto $J/\psi \rightarrow \mu^+\mu^-$ and $\psi(2S) \rightarrow \mu^+\mu^-$ decays; they are 2.796–3.216 GeV/c$^2$ and 3.436–3.806 GeV/c$^2$, respectively.
Figure 3: Distribution of the pion DLL($\pi$-$K$) versus the $\pi^+\pi^-\mu^+\mu^-$ mass; for each candidate, the minimum value between the DLL($\pi$-$K$) of the two pions is considered. The $B^0 \to J/\psi \pi^+\pi^-$ and $B^0 \to \pi^+\pi^-\mu^+\mu^-$ candidates are merged together. The variable DLL($\pi$-$K$) is the difference of the logarithm of the likelihood of a track to be a $\pi$ or a $K$. The accumulation of events at DLL($\pi$-$K$)= 0 is given by particles with momentum below the Cherenkov threshold in the RICH detectors. The accumulation of candidates in the region around 5.2 GeV/$c^2$ for DLL($\pi$-$K$)< 1 is due to $B^0 \to J/\psi K^*(892)^0$ and $B^0 \to K^*(892)^0 \mu^+\mu^-$ decays, where the kaon is misidentified with a pion. The band around 5.37 GeV/$c^2$ is given by $B^0 \to J/\psi \pi^+\pi^-$ and $B^0 \to \pi^+\pi^-\mu^+\mu^-$ decays candidates; the band around 5.28 GeV/$c^2$ is due to $B^0 \to J/\psi \pi^+\pi^-$ and $B^0 \to \pi^+\pi^-\mu^+\mu^-$ decays candidates. In the analysis, the requirement DLL($\pi$-$K$)> 1 is set to suppress the misidentified decays.
Figure 4: Distribution of the $\pi^+\pi^-$ mass versus the $\pi^+\pi^-\mu^+\mu^-$ mass. The $B^0_{(s)} \to J/\psi \pi^+\pi^-$ and $B^0_{(s)} \to \pi^+\pi^-\mu^+\mu^-$ candidates are merged together.
Figure 5: Selection efficiency for $B_s^0 \rightarrow \pi^+\pi^-\mu^+\mu^-$ decays as a function of the $\pi^+\pi^-$ mass for simulated events. The black curve represents the fit to the simulated data with a second-order polynomial. The variation of the efficiency is of the order of 15%.

Figure 6: Likelihood profiles of (a) $R_d$ and (b) $R_s$. For each value of $R_d$ ($R_s$) probed, all other parameters are floating in the minimisation of the likelihood. Systematic uncertainties are not included.
Figure 7: Background-subtracted $\pi^+\pi^-$ mass distributions of $B^{0}_{(s)} \to J/\psi \pi^+\pi^-$ candidates.