The $B_c^+$ meson is the only ground-state meson consisting of two heavy quarks of different flavour, namely a $b$ and a $c$ quark. As such, its formation in $pp$ collisions is suppressed relative to the lighter $B$ mesons. Unlike $B^0$, $B^+$, and $B_d^0$ mesons, the $b$-quark decay accounts for only $\sim 20\%$ of the $B_c^+$ width \cite{1}. Around $70\%$ of its width is due to $c$-quark decays, where the $c$-quark transition has been observed with $B_c^+ \to B_s^0 \pi^+$ decays \cite{2}. This leaves $\sim 10\%$ for $\bar{b}c \to W^+ \to q\bar{q}$ annihilation amplitudes, which can be unambiguously probed in charmed final states. No charmless $B_c^+$ decays have been reported to date, although searches show an indication at the level of $2\%$ standard deviations ($\sigma$) \cite{3}.

To test QCD factorisation and explore the new physics potential of $B_c^+$ decays, rarer decays such as suppressed tree-level $b \to u$ transitions and $b \to s$ loop-mediated (penguin) decays can be studied, where the charm quantum number remains unchanged. The simplest decay is the colour-allowed $B_c^+ \to D^{(*)0}\pi^+$ decay, illustrated in Fig. 1(a). The expected branching fraction for this decay is a factor $[V_{ub}/V_{cb}]^2 \approx 0.007$ lower than the favoured $b \to c$ and colour-allowed $B_c^+ \to J/\psi \pi^+$ decay \cite{4,5}, placing this mode at the limit of sensitivity with current LHCb data. However, this expectation may be enhanced by penguin and weak annihilation amplitudes, which will be more pronounced in the $B_c^+ \to D^{(*)0}K^+$ mode (see Fig. 1(b,c)). This motivates a search for the $B_c^+ \to D^{(*)0}K^+$ and $B_c^+ \to D^{(*)0}\pi^+$ decays, particularly as the branching fraction estimates in the literature vary considerably \cite{6-8}.

The decay $B^+ \to D^{0}\pi^+$ is used for normalisation. Since the ratio of production rates for $B_c^+$ and $B^+$ mesons within the LHCb acceptance, $f_c/f_u$, is unknown, the measured observables are

$$R_{D^{(*)0}h} = \frac{f_c}{f_u} \times \mathcal{B}(B_c^+ \to D^{(*)0}h^+),$$

where $h$ is $\pi$ or $K$ and $\mathcal{B}(B_c^+ \to D^{(*)0}h^+)$ represents the corresponding branching fraction. Separate observables are thus measured for the $B_c^+ \to D^0\pi^+$, $B_c^+ \to D^0K^+$, $B_c^+ \to D^{(*)0}\pi^+$ and $B_c^+ \to D^{(*)0}K^+$ decays. Theoretical estimates for $\mathcal{B}(B_c^+ \to J/\psi\pi^+)$ range from $6.0 \times 10^{-4}$ [9] to $1.8 \times 10^{-3}$ [10], which implies $f_c/f_u$ values in the range

![Figure 1](image-url)

Figure 1: Tree (a), penguin (b), and weak annihilation (c) diagrams for the decays studied. In each case, the meson appearing before the comma denotes the favoured decay.