1 Distribution of flattened MLP response

The MLP response distributions for the signal, background and same-sign samples, after being transformed to make the signal candidates distributed evenly between zero and unity, are shown in Fig. 1.

![Flattened MLP response](image)

Figure 1: Flattened distribution of the MLP response for the signal sample from simulation, together with the distributions for the background from the sidebands and same-sign samples. The red line stands for the signal simulation, the black line the data sidebands, and the blue line the same-sign sample. The vertical green lines indicate the boundaries of the MLP categories.

2 Theoretical predictions of $R$

The ratio $R$ can be predicted using BCVEGPY [1]. The wave function at the origin, $R(0)$, is taken to be $1.241 \text{ GeV}^{3/2}$ for $B_c(1S)$ states, and $0.991 \text{ GeV}^{3/2}$ for $B_c(2S)$ states [2]. The masses of $b$ and $c$ quarks are set to $m(b) = 5400 \text{ MeV}/c^2$ and $m(c) = 1458 \text{ MeV}/c^2$ for $B_c(2S)^+$, and $m(b) = 5400 \text{ MeV}/c^2$ and $m(c) = 1490 \text{ MeV}/c^2$ for $B_c^*(2S)^+$, respectively. The production cross-sections of the $B_c$ mesons are calculated using several theoretical models [3,4]. Under the assumption that 15% of the $B_c^+$ mesons come from the $P$-wave states [3], the BCVEGPY generator predicts

$$\frac{\sigma_{B_c(2S)^+}}{\sigma_{B_c^*}} = 0.04$$  \hspace{1cm} (1)
and
\[ \frac{\sigma_{B_c^+(2S)^+}}{\sigma_{B_c^+}} = 0.10, \] (2)

which are consistent with the predictions given in Ref. [5], while according to Ref. [6], the production cross-section ratios are
\[ \frac{\sigma_{B_c^+(2S)^+}}{\sigma_{B_c^+}} = 0.09 \] (3)

and
\[ \frac{\sigma_{B_c^+(2S)^+}}{\sigma_{B_c^+}} = 0.23. \] (4)

Considering the branching fractions \( \mathcal{B}(B_c^+(2S)^+ \rightarrow B^{(s)+}_c \pi^+ \pi^-) \), Ref. [5] predicts \( \mathcal{B}(B_c^+(2S)^+ \rightarrow B^+_c \pi^+ \pi^-) = 49\% \) and \( \mathcal{B}(B_c^+(2S)^+ \rightarrow B^{(s)+}_c \rightarrow B^+_c \gamma) \pi^+ \pi^-) = 39\% \), and Ref. [7] predicts \( \mathcal{B}(B_c^+(2S)^+ \rightarrow B^+_c \pi^+ \pi^-) = 59\% \) and \( \mathcal{B}(B_c^+(2S)^+ \rightarrow B^{(s)+}_c \rightarrow B^+_c \gamma) \pi^+ \pi^-) = 53\% \). The predicted values of \( \mathcal{R} \) are summarised in Table 1.

<table>
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<th>Ref. for ( \mathcal{B} ) prediction</th>
<th>( \mathcal{R}_{B_c(2S)^+} )</th>
<th>( \mathcal{R}_{B_c^+(2S)^+} )</th>
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References


